Electrodinàmica clàssica

pod

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$\vec{r'}_\perp = \vec{r}_\perp\,, \qquad r'_\parallel = \gamma(r_\parallel - vt)\,, \qquad t' = \gamma(t - \frac{1}{c}\vec{r}\vec{\beta})$
$x' = x\cosh\xi - ct\sinh\xi\,, \qquad ct' = ct\cosh\xi - x\sinh\xi$
$\vec{r'} = \vec{r} + \left(\frac{\gamma-1}{v^2}\vec{r}\cdot\vec{v} - \gamma t\right)\vec{v}\,, \qquad t' = \gamma\left(t - \frac{\vec{v}\cdot\vec{r}}{c^2}\right)$
$i,j \le 3 \;\; L^{i'}_{j} = \delta^{i'}_{j} + \frac{\gamma-1}{\beta^2}\beta^{i'}\beta_j$
$i \le 3 \;\; L^{i'}_{4} = L^{4'}_{i} = -\gamma\beta^i$
$L^{4'}_{4} = \gamma$

Contracció longituds: $\Delta\vec{x'}_\perp = \Delta\vec{x}_\perp\,, \qquad \Delta x'_\parallel = \gamma\Delta x_\parallel$
$l = l'\sqrt{1 - \left(\frac{v}{c}\right)^2\cos^2\alpha'} = \frac{l'}{\sqrt{\gamma^2\cos^2\alpha + \sin^2\alpha}}\,, \; \mathrm{tg}\alpha' = \frac{1}{\gamma}\mathrm{tg}\alpha$
Temps: $\Delta t = \gamma\Delta t' = \gamma\tau$

Velocitats: $\vec{u'}_\perp = \frac{\vec{u}_\perp}{\gamma\left(1 - \frac{\vec{v}\cdot\vec{u}}{c^2}\right)}\,, \quad u'_\parallel = \frac{u_\parallel - v}{1 - \frac{\vec{u}\cdot\vec{v}}{c^2}}\,, \quad \phi' = \phi - \xi$
$\vec{u'} = \frac{\vec{u} + \frac{\gamma-1}{v^2}\vec{u}\cdot\vec{v} - \gamma\vec{v}}{\gamma\left(1 - \frac{\vec{v}\cdot\vec{u}}{c^2}\right)}\,, \quad \mathrm{tg}\theta' = \frac{u\sin\theta}{\gamma\left(u\cos\theta - v\right)}\,, \quad v = c\,\mathrm{tgh}\xi$
$u' = \frac{\sqrt{u^2 + v^2 - 2uv\cos\theta - \left(\frac{vu}{c}\sin\theta\right)^2}}{1 - \frac{\vec{v}\cdot\vec{u}}{c^2}}\,, \quad \gamma'_u = \gamma_u\gamma_v\left(1 - \frac{uv}{c^2}\right)$

Ones planes: $\vec{n} = \vec{k}/2\pi = \hat{n}/\lambda$
$\vec{n'} = \vec{n} + \frac{\gamma-1}{c^2}(\vec{v}\cdot\vec{n})\vec{v} - \frac{\gamma\nu}{c^2}\vec{v}\,, \quad \nu' = \gamma(\nu - \vec{v}\cdot\vec{n})$
Doppler: $\nu_R = \nu_E \Big/ \gamma\left(1 - \frac{\vec{v}\cdot\hat{n}_R}{c_{1,R}}\right)$
D. rad (lluny): $\nu_R = \nu_E\sqrt{\frac{1-v/c}{1+v/c}}$, tg: $\nu_R = \nu_E/\gamma$
Aberració: $\sin\alpha' = \frac{\sin\alpha}{\gamma(1+\beta\cos\alpha)}\,, \qquad \cos\alpha' = \frac{\cos\alpha + \beta}{1 + \beta\cos\alpha}$
Fresnel: $c'_1 = c_1\frac{1 - \frac{v}{c_1}\cos\alpha}{\sqrt{1 - 2\cos\alpha\frac{vc_1}{c^2} - \left(\frac{v}{c}\sin\alpha\right)^2}} \approx c_1 - v\left(1 - \frac{1}{r_1^2}\right)$

Quadrivectors: $v^{\mu'} = \Lambda^{\mu'}_{\nu}v^\nu\,, \; \Lambda = L\cdot R$

t. propi $\Delta\tau = \gamma^{-1}\Delta t = \frac{1}{c}\int_{\lambda_0}^{\lambda}\mathrm{d}\lambda\sqrt{-\dot{X}^\mu\dot{X}_\mu}$
4-vel: $u^\mu = \frac{\mathrm{d}X^\mu}{\mathrm{d}\tau} = \gamma(\vec{\omega}, c)\,, \; (u)^2 = -c^2\,, \; \vec{\beta} = \frac{\vec{\omega}}{c} = \frac{\vec{u}}{u^4}$
4-acc: $b^\mu = \frac{\mathrm{d}u^\mu}{\mathrm{d}\tau}\,, \; b^4 = \gamma^4\frac{\vec{\omega}\cdot\vec{a}}{c} = \frac{\vec{b}\cdot\vec{\omega}}{c} = \frac{\vec{u}\cdot\vec{b}}{u^4}\,, \; b^\mu u_\mu = 0$
$\vec{b} = \gamma^2\vec{a} + \gamma^4\left(\frac{\vec{\omega}\cdot\vec{a}}{c^2}\right)\vec{\omega} = \gamma^4\left(\vec{a} + \frac{1}{c^2}\vec{\omega}\times(\vec{a}\times\vec{\omega})\right)$
$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = \frac{1}{c^2}\vec{\omega}\cdot\vec{a}\gamma^3\,, \quad b^\mu b_\mu = \gamma^4\left(\vec{a}^2 + \gamma^4\frac{\vec{\omega}\cdot\vec{a}}{c^2}\right)$

4-p: $p^\mu = mu^\mu = m\gamma(\vec{\omega}, c)\,, \; p^\mu p_\mu = -m^2c^2\,, \; E = cp^4$
$E^2 = (c\vec{p})^2 + (mc^2)^2\,, \; \vec{\beta} = \vec{p}/p^4\,, \; c|\vec{p}| = \sqrt{T(T + 2mc^2)}$
$\vec{p}_{cm} = 0\,, \; \vec{v}_{cm} = c\vec{p}/p^4\,, \qquad \vec{K} = h\nu/c(\hat{n}, 1)$

Dinàmica: $f^\mu = \frac{\mathrm{d}}{\mathrm{d}\tau}p^\mu = \gamma(\vec{F}, \vec{v}\vec{F}/c)$
$\vec{F} = m\gamma\vec{a} + m\gamma^3\frac{\vec{v}\cdot\vec{a}}{c^2}\vec{v} \longrightarrow_{(\vec{v}\perp\vec{a})} m\gamma^3\vec{a}$

$L = -mc^2\sqrt{1 - \frac{v^2}{c^2}}\,, \; H = c\sqrt{m^2c^2 + \vec{p}^2}$
Acció $S = \int L\mathrm{d}t = -mc^2\int\mathrm{d}\tau$

$\vec{\nabla}\cdot\vec{B} = 0\,, \quad \vec{\nabla}\times\vec{E} + \partial_t\vec{B} = 0\,, \quad \vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon\vec{E}$
$\vec{\nabla}\cdot\vec{D} = \rho\,, \quad \vec{\nabla}\times\vec{H} - \partial_t\vec{D} = \vec{\jmath}\,, \quad \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \mu\vec{B}$
4-corrent: $j^\mu = (\vec{\jmath}, c\rho)\,, \quad$ Continuïtat: $\frac{\partial}{\partial x^\mu}j^\mu = 0$
Força de Lorentz $f^\mu = qF^\mu{}_\nu u^\nu$
Tensor electromagnètic $F_{i4} = -F_{4i} = E_i/c$
$F_{ij} = \epsilon_{ijk}B^k\;; \quad E_i = cF_{i4} = -cF_{4i}\,, \quad B^k = \frac{1}{2}\epsilon^{ijk}F_{ij}$
Camp E: $E'_\parallel = E_\parallel\,, \; \vec{E}'_\perp = \gamma\left(\vec{E}_\perp + \vec{v}\times\vec{B}\right)$
Camp B: $B'_\parallel = B_\parallel\,, \; \vec{B}'_\perp = \gamma\left(\vec{B}_\perp - \frac{1}{c^2}\vec{v}\times\vec{E}\right)$
$\rightarrow \vec{E}' = \vec{E} + \gamma(\vec{v}\times\vec{B}) + \frac{1-\gamma}{v^2}\vec{v}\times(\vec{v}\times\vec{E})$
$\rightarrow \vec{B}' = \vec{B} - \frac{\gamma}{c^2}(\vec{v}\times\vec{E}) + \frac{1-\gamma}{v^2}\vec{v}\times(\vec{v}\times\vec{B})$
Invariants $\vec{E}\cdot\vec{B} \quad , \quad \vec{B}^2 - \frac{1}{c^2}\vec{E}^2$

Maxwell $\partial_\rho F^{\nu\rho} = \mu_0 j^\nu$, $\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0$
4-potecial $A^\nu = (\vec{A}, \phi/c)$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$\rightarrow$ Maxwell $\partial^\nu \partial_\rho A^\rho - \partial_\rho \partial^\rho A^\nu = \mu_0 j^\nu$
Galga Coulomb $\vec{\nabla} \cdot \vec{A} = 0$
Lorentz $\partial_\mu A^\mu = 0$

Energía $\vec{\nabla} \cdot \vec{S} + \frac{\partial U}{\partial t} = -\vec{\jmath} \cdot \vec{E}$, $\vec{S} = \vec{E} \times \vec{H}$
Energia-impuls $\theta^{\mu\alpha} = \epsilon_0 c^2 \left(F^{\nu\mu} F^\alpha_\nu - \frac{\eta}{4} F^{\sigma\rho} F_{\sigma\rho}\right)$
$\theta^{44} = U$, $\theta^{i4} = \theta^{4i} = S_i/c$
$\theta^{ij} = -\epsilon_0 \left[E^i E^j + c^2 B^i B^j - \frac{1}{2}\delta^{ij}(\vec{E}^2 + c^2 \vec{B}^2)\right] = -T^{ij}$
Conservació $\partial_\mu \theta^{\mu\alpha} = j_\nu F^{\nu\alpha}$
Moment $\frac{\mathrm{d}}{\mathrm{d}t}\int_v (\rho E_i + (\vec{\jmath} \times \vec{B})_i) \mathrm{d}v = -\frac{1}{c^2}\frac{\mathrm{d}}{\mathrm{d}t}\int_v S_i \mathrm{d}v + \int_A T^{ij} \hat{n}_i \mathrm{d}^2 A$

$L(\vec{x}, t) = -mc^2\sqrt{1 - \frac{v^2}{c^2}} + q(\vec{v} \times \vec{A}) - q\phi$
$P_i = \frac{\partial L}{\partial v^i} = m\gamma v_i + qA_i$
$H(P, \vec{x}, t) = \sqrt{m^2 c^4 + c^2(\vec{P} - q\vec{A})^2} + q\phi$

Telegrafia $\nabla^2 \vec{E} - \mu\epsilon \frac{\partial^2}{\partial t^2}\vec{E} - \sigma\frac{\partial}{\partial t}\vec{E} = 0$
si $\vec{E} = \vec{E}(\vec{r})\mathrm{e}^{-i\omega t} \rightarrow \nabla^2 \vec{E} + \left(1 + \frac{\sigma}{\mu\epsilon}\right)\mu\epsilon\vec{E} = 0$
Transversalitat $c_1 \vec{B} = \hat{n} \times \vec{E}$
Temps retardat $\tau_r = t - \frac{1}{c}|\vec{x} - \vec{y}|$, $c^2(t - t_r)^2 - (\vec{x} - \vec{z}_r)^2 = 0$

Radiació multipolar
$A^\mu(\vec{x}, t) = \frac{\mu_0}{4\pi}\int_{R^3} \mathrm{d}^3\vec{y}\frac{j^\mu\left(\vec{y}, t \mp \frac{|\vec{y} - \vec{x}|}{c}\right)}{|\vec{x} - \vec{y}|}\theta\left(\pm t - \frac{|\vec{x} - \vec{y}|}{c}\right) \pm$
$\pm\frac{1}{4\pi}\int_{R^3}\frac{\mathrm{d}^3\vec{y}}{|\vec{x} - \vec{y}|}\left[\frac{1}{c}\partial_t A^\mu(\vec{y}, 0)\delta(ct \mp |\vec{x} - \vec{y}|) + \right.$
$\left. + A^\mu(\vec{y}, 0)\delta'(ct \mp |\vec{x} - \vec{y}|)\right]$
Part. lliures a $t \rightarrow -\infty$
$\rightarrow A^\mu = \frac{\mu_0}{4\pi}\int_{R^3} \mathrm{d}^3\vec{y}\frac{j^\mu\left(\vec{y}, t \mp \frac{|\vec{y} - \vec{x}|}{c}\right)}{|\vec{x} - \vec{y}|}$

Font localitzada. Camps pròxims
$\vec{E}_I = \frac{1}{4\pi\epsilon_0}\int_{\mathbb{R}^3} \mathrm{d}^3\vec{y}\frac{\rho(\vec{y})}{|x - y|^2}\mathrm{e}^{-ik|\vec{x} - \vec{y}|}$
$\vec{B}_I = -\frac{\mu_0}{4\pi}\int_{\mathbb{R}^3} \mathrm{d}^3\vec{y}\frac{\hat{n} \times \vec{\jmath}(\vec{y})}{|\vec{x} - \vec{y}|^2}\mathrm{e}^{-ik|\vec{x} - \vec{y}|}$
Camps de radiació. Camps de radiació
$\vec{E}_{II} = \frac{ik}{4\pi\epsilon_0}\int_{\mathbb{R}^3} \mathrm{d}^3\vec{y}\frac{\mathrm{e}^{-ik|\vec{x} - \vec{y}|}}{|\vec{x} - \vec{y}|}(\rho\hat{n} - \frac{1}{c}\vec{\jmath}(\vec{y})) = i\omega\hat{r} \times (\hat{r} \times \vec{A})$
$\vec{B}_{II} = -\frac{ik\mu_0}{4\pi}\int_{\mathbb{R}^3} \mathrm{d}^3\vec{y}\frac{\mathrm{e}^{-ik|\vec{x} - \vec{y}|}}{|\vec{x} - \vec{y}|}\hat{n} \times \vec{\jmath}(\vec{y})) = -\frac{i\omega}{c}\hat{r} \times \vec{A}$
$\vec{A} = \frac{\mu_0}{4\pi}\frac{\mathrm{e}^{-ikr}}{r}\int_V \mathrm{d}^3\vec{y}\mathrm{e}^{ik\hat{r}\vec{y}}\vec{\jmath}(\vec{y})$

Radiació dipolar magnètica ($\mathrm{e}^{ik\hat{r}\vec{y}} \approx 1$)
$\vec{A} = i\frac{\mu_0\omega}{4\pi}\frac{\mathrm{e}^{-ikr}}{r}\vec{p}$, $\vec{p} = \int_v \mathrm{d}^3\vec{r}\rho(\vec{y})\vec{y}$
Moment dipolar magnètic (part antisimétrica)
$\vec{A} = -\frac{ik\mu_0}{4\pi}\frac{\mathrm{e}^{-ikr}}{r}\hat{r} \times \vec{m}$, $\vec{m} = \int_v \mathrm{d}^3\vec{y}\ \vec{M} = \int_v \mathrm{d}^3\vec{y}\ \frac{1}{2}\vec{y} \times \vec{\jmath}(\vec{y})$
Moment quadrupolar elèctric (part simètrica)
$\vec{B} = -i\frac{ck^2}{24\pi}\mu_0\frac{\mathrm{e}^{-ikr}}{r}\hat{r} \times \vec{q}$, $\vec{E} = -i\frac{c^2 k^3}{24\pi}\mu_0\frac{\mathrm{e}^{-ikr}}{r}(\hat{r} \times \vec{q}) \times \hat{r}$
$a^{ij} = \int_v \mathrm{d}^3\vec{y}(3y^i y^j - \vec{y}^2\delta^{ij})\rho(\vec{y})$, $q^j = \hat{r}_i a^{ij}$
$\hat{r} \times \vec{q} = 3\hat{r} \times \int_v \mathrm{d}^3\vec{y}(\vec{y}\hat{r})\vec{y}\rho(\vec{y})$

Partícules en moviment $j^\mu = q\delta(\vec{y} - \vec{z}(t))(\vec{v}(t), c)$, $R^\rho = x^\rho - y^\rho$, $\rho_r = |\vec{x} - \vec{z}|$
$A^\nu(\vec{x}, t_r) = -\frac{\mu_0 c}{4\pi}q\frac{\dot{z}^n u}{(\vec{x} - \vec{z})^\rho \dot{z}_\rho}$
$F_I^{\mu\nu} = -\frac{q}{4\pi\epsilon_0 c}\frac{\dot{z}^\mu r^\nu - \dot{z}^\nu r^\mu}{\rho_r^2}$
$F_{II}^{\mu\nu} = -\frac{q}{4\pi\epsilon_0 c}\left\{\frac{\ddot{z}^\mu r^\nu - \ddot{z}^\nu r^\mu}{c\rho_r} + \frac{\dot{z}^\mu r^\nu - \dot{z}^\nu r^\mu}{c\rho_r}(r\ddot{z})\right\}$
a $t, \tau_r \rightarrow -\infty$, $\ddot{z} = 0 \rightarrow F_{II} = 0$

$$\vec{E}_{II} = \frac{q}{4\pi\epsilon_0 c|\vec{x} - \vec{z}|(1 - \hat{n}\vec{\beta})^3}\hat{n} \times (\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}$$

$$\vec{B}_{II} = \frac{q}{4\pi\epsilon_0 c^3|\vec{x} - \vec{z}|(1 - \hat{n}\vec{\beta})}\Big((1 - \vec{\beta}\hat{n})(\vec{a} \times \hat{n}) + (\vec{a}\hat{n})(\vec{\beta} \times \hat{n})\Big) = \frac{1}{c}\hat{n} \times \vec{E}_{II}$$

4-moment radiat $\frac{\mathrm{d}}{\mathrm{d}\tau}p^\mu = \frac{q^2}{6\pi\epsilon_0 c^5}(\ddot{z}^\nu \ddot{z}_\nu)\dot{z}^\mu$
Energía: $\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon = \frac{q\gamma^6}{6\pi\epsilon c^3}(\vec{a}^2 - (\vec{\beta} \times \vec{a})^2)$
formula de Larmor $\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon = \frac{q^2}{6\pi\epsilon_0 c^3}\vec{a}^2$

Distribució angular $\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon = \vec{S}\mathrm{d}^2\vec{A}$
observador $\frac{\mathrm{d}^2\epsilon}{\mathrm{d}t\mathrm{d}^2\Omega} = \frac{q^2}{16\pi^2\epsilon_0 c}\frac{1}{(1 - \hat{n}\vec{\beta})^6}\left(\hat{n} \times \left((\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}\right)\right)^2$ càrrega (t. retardat)

$$\frac{\mathrm{d}^2\epsilon}{\mathrm{d}t\mathrm{d}^2\Omega} = \frac{q^2}{16\pi^2\epsilon_0 c}\frac{1}{(1 - \hat{n}\vec{\beta})^5}\left(\hat{n} \times \left((\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}\right)\right)^2$$

Accelerador lineal $\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon = \frac{q^2\gamma^6}{6\pi\epsilon_0 c^3}\vec{a}^2$, $\frac{\mathrm{d}}{\mathrm{d}t}\varepsilon = m\gamma^3\vec{v}\vec{a}$
$\frac{\mathrm{d}\varepsilon_{rad}}{\mathrm{d}\varepsilon} = \frac{q^2}{6\pi\epsilon_0 c^3 m^2 v}\frac{\mathrm{d}\varepsilon}{\mathrm{d}x}$, $\mathrm{d}\varepsilon = m\gamma^3 va\mathrm{d}t = m\gamma^3 a\mathrm{d}x$
Distribució angular $\frac{\mathrm{d}^2 W}{\mathrm{d}\Omega^2} = \frac{q^2\dot{v}^2}{16\pi^2\epsilon_0 c^3}\frac{\sin^2\theta}{(1 - \beta\cos\theta)^5}$
$f(\theta) = \frac{3(1 - \beta^2)}{8\pi}\frac{\sin^2\theta}{(1 - \beta\cos\theta)^5}$, $\cos\theta_{\max} = \frac{-1 + \sqrt{1 + 15\beta^2}}{3\beta}$
resultats $\frac{\mathrm{d}^2 W}{\mathrm{d}\Omega^2} \propto \gamma^8$, $\Delta\theta \propto \gamma^{-1}$
triem $\vec{\beta} = \beta\hat{k}$, $\hat{n} = (\sin\theta\cos\varphi, \sin\theta\cos\varphi, \sin\varphi)$

Accelerador circular $\frac{\mathrm{d}}{\mathrm{d}t}\omega = \frac{q^2\gamma^4}{6\pi\epsilon_0 c^3}a^2$

$\frac{\Delta\varepsilon_{rad,1rev}}{\varepsilon} = \frac{\Delta\varepsilon}{m\gamma c^2} = \frac{q^2\epsilon^2\beta^3}{3\epsilon_0 R m^4 c^8}$

$\frac{\mathrm{d}^2W}{\mathrm{d}\Omega^2} = \frac{q^2\dot{\beta}^2}{16\pi^2\epsilon_0 c(1-\beta\cos\theta)^5}\left[(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^\theta\cos^\varphi\right]$

$\rho = \frac{3}{8\pi\gamma^4(1-\beta\cos\theta)^3}\left[1 - \frac{\sin^2\theta\cos^2\varphi}{\gamma^2(1-\beta\cos\theta)^2}\right]$

si$\gamma\uparrow\rightarrow\rho\approx\frac{1}{(1+\gamma^2\theta^2)}\left(1-\gamma^2\frac{\theta^2\cos^2\varphi}{(1+\gamma^2\theta^2)^2}\right)$

triem $\vec{\beta} = \beta\hat{k}$, $\quad \dot{\vec{\beta}} = \dot{\beta}\hat{\imath}$

Sincrotó $\delta t = 2\delta\theta R(1/\beta - 1 =\approx \gamma^{-3}/\omega_0$

Radi de radiació $\delta\omega \approx \omega_0\gamma^3$